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Thank you.

creative Maths Problems

**Awakening Confidence
and Enthusiasm
Book B**

Algebra, Statistics and Logic

Nicolla Hansen



Published By User Friendly Resources. Book No. 485B



TITLE

Book Name: Creative Maths Problems
– Awakening confidence and enthusiasm
Book Number: 485B
ISBN Number: 1-86968-288-2
Published: October, 2006

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ACKNOWLEDGEMENTS

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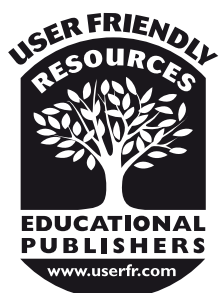
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Introduction

My motivation for developing this resource stems from a personal ambition to help students experience the joy and satisfaction that can come from mathematical problem solving. Through observation and experimentation in the maths classroom, I believe I have gained some insight into strategies that work and strategies that do not work. Above all it seems to me that the recipe for success is providing students with work that they perceive as achievable and fun. I hope that the problems in this book will be a source of motivation for students, encouraging them to problem solve with confidence and enthusiasm.

In developing this resource, I was conscious of the fact that mathematic education places a huge emphasis on students demonstrating sound mathematical process in their approach to problems. Teachers are being encouraged to teach students to think logically, apply reason and communicate effectively. Mathematical problem solving has the potential to teach these skills, but in my opinion, educators need to proceed with some caution. Too often I have seen students become frustrated and disillusioned with maths problems because they lack the skills and the confidence to solve them. It is my hope that this book will assist students in acquiring the skills necessary to confidently investigate any mathematical problem they are presented with. In particular the suggestions for teachers, which accompany each problem, are designed to help teachers teach students to approach the problems confidently, strategise appropriately and gain a deeper understanding of the mathematical process involved.

There are also opportunities for students to engage in cooperative learning, pose new problems for investigation or enter into classroom discussion and debate. These are generic skills that today's society is demanding from young people leaving the education system.

The problems in this book have been organised into three areas within mathematics:

- Algebra
- Statistics
- Logic



The “star” key is intended as a quick way for teachers to gauge the appropriateness of a particular problem. It may also be used as a motivational tool for students:

Key to the difficulty of problems:

- ☆ Suitable for students who are stars in the making
- ☆☆ Suitable for students currently on the road to stardom
- ☆☆☆ Suitable for students who have achieved shooting star status

The problems have the potential to be used in a variety of ways depending on the learning needs of the students. Some suggestions are:

- As lesson starters presented on an overhead projector.
- As a supplement or extension to work being taught.
- As the basis for cooperative learning activities in the classroom.
- As an incentive for fast finishers in the class.
- As challenging homework assignments.
- As a “fill in” for the last five minutes of a lesson.
- As the basis for a school competition (e.g. “maths puzzle of the week”).

At the end of the day, how you use this resource is up to you. However, the underlying objective should always be to increase participation and enthusiasm in mathematics. In particular, I hope that you will share in my goal of teaching students to become confident mathematical problem solvers who have taken on an “I can do” attitude.



Vertical Vertigo

The tallest structure in the Southern Hemisphere is Auckland's Sky Tower, standing 328 metres tall. There are 1029 steps from the base of the Sky Tower to the main observation level. During a special event known as the Sky Tower Vertical Challenge, these steps are open to the public.

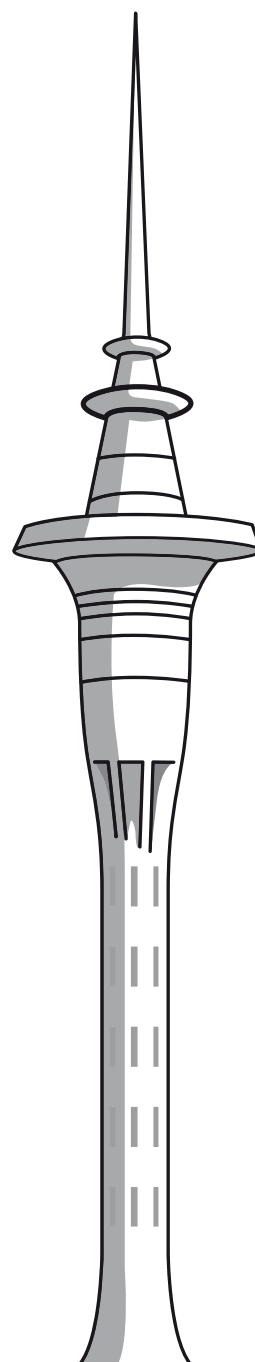


http://www.skycityauckland.co.nz/skycity/auckland/sky-tower/sky-tower_home.cfm

Problem

One year Jim and Phillipa decided to enter the vertical challenge. Phillipa, a keen athlete, ran up the 1029 steps in 11 minutes and 40 seconds. Jim, who was a little out of shape, decided to catch the elevator to the main observation level and walk down rather than up! Walking down took him 18 minutes and 52 seconds.

If Phillipa and Jim took their first step at exactly the same time (Jim from the top, Phillipa from the bottom) and each of them maintained an even pace the whole way, on which step would you expect them to pass each other?





Gifts Galore

'The 12 Days of Christmas' is a well-known song and talks about the gifts received on each of the twelve days following Christmas. As the song goes, I received a partridge in a pear tree on the first day of Christmas. On the second day I was given two turtle doves and a partridge in a pear tree. On the third day I was presented with three French hens, two turtle doves and a partridge in a pear tree. This pattern continues for all twelve days.

In order the gifts I received were: a partridge in a pear tree, turtle doves, French hens, calling birds, gold rings, geese laying, swans swimming, maids milking, ladies dancing, lords leaping, pipers piping and drummers drumming.



Problem

Of all the gifts that I received from my true love, which gift did I receive the most of?



Money Matters

The word 'dollar' has its origins in the Roman Empire. The official source of coinage for the entire Holy Roman Empire was from a silver mine in the mountains of Bohemia. The coins were named 'Joachimstalers' after the Joachimsthal Valley from which they were mined. Over time Joachimstalers shortened to 'Talers' and later, due to American pronunciation, became widely accepted as 'dollars'.



<http://www.didyouknow.cd/dollar.htm>



Problem

John said to his four daughters, "The girl who is closest to guessing how many dollars I have in my pocket can keep them".

When John looked at the four guesses he found the following:

His eldest daughter's guess was \$10 too much.

His second daughter's guess was \$5 short.

His third daughter's guess was \$7 under.

His youngest daughter's guess was \$8 over.

If the four guesses added up to \$86, how much money was in John's pocket?



Catty Conundrum

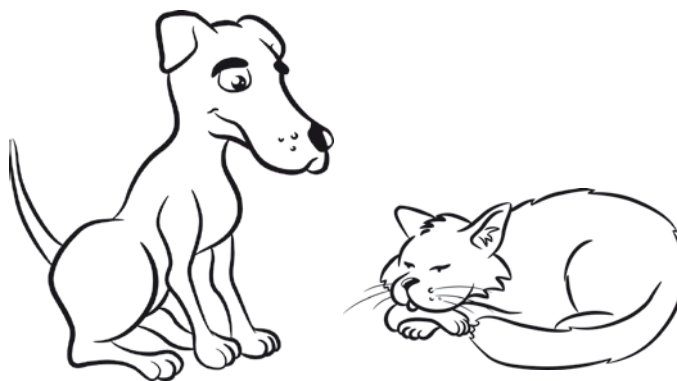
Have you ever wondered how mature your cat or dog really is? According to studies, a one year-old cat is equivalent in age to a 15 year-old human. A two-year-old cat is equivalent to a 24 year-old human. After the age of two, add four years for each successive year of your cat's life to get his human age equivalent. The general rule of thumb for dogs is slightly easier. Simply multiply your dog's age by seven to get his human age equivalent.



<http://www.coolmath4kids.com/calculators/dogyears.html>



<http://cats.about.com/cs/catmanagement101/ht/convertcatyears.htm>



Problem

Russell was 23 years old when his dog was born and 22 years old when his cat was born. Russell's dog is now equivalent in age to a 28 year old human. What human age is his cat equivalent to?



Can It

A Frenchman by the name of Nicolas Appert is credited with discovering that food remained fresh if sealed tightly in a can or jars and heated. However it took 48 years, after the invention of the tin can for can openers to be invented! Prior to that, canned food came with a label stating; 'cut around on the top near to outer edge with a chisel and hammer'.

 <http://www.didyouknow.cd/cans.htm>



Problem

Barbara is very pedantic when it comes to organising the tin cans in her pantry. All her food cans are organised in perfectly straight rows, each containing a different food. Each row also contains one more can than the row before. From front to back, the rows of cans are as follows: asparagus, peaches, corn, tomatoes and apricots.

If Barbara has 45 food cans in total, how many cans of apricots does she have?



Give it a Try

Rugby is played in over 100 different countries. Players may handle the oval shaped ball or run with it but are only allowed to pass the ball backwards. The best way to earn points is to score a try, which involves grounding the ball behind the other team's goal line. When this happens the scoring team earns five points and the right to attempt a conversion kick. If the kick successfully sails through the upright posts, a further two points is awarded. Teams can also earn points by kicking penalties (three points) or drop goals (three points).



Problem

Australia and New Zealand, two of the greatest rugby nations in the world, were playing a rugby match. At the end of the game, Australian fans were thrilled. Australia had won the match with a final score of 21.

How many different ways could Australia have scored 21 points?



Biking Bliss

About a billion bicycles exist in the world and approximately 400 million are in China. The fastest speed ever recorded by someone riding a bicycle was 247 km/hour. An American man, John Howard, who rode in the slipstream of a specifically designed car, achieved this speed in 1985.

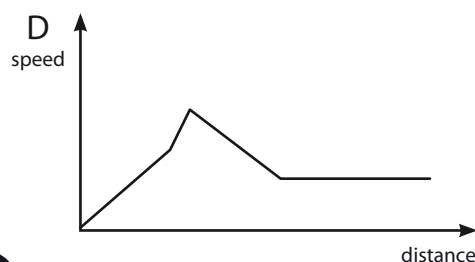
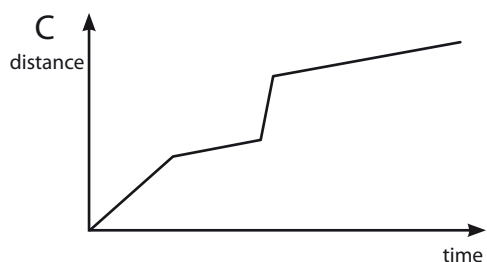
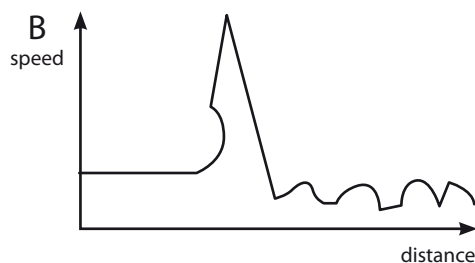
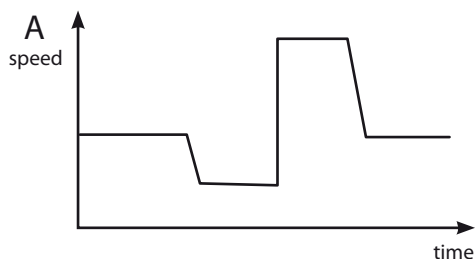


<http://www.didyouknow.cd/bicycles.htm>

Problem

Maggie went for a bike ride. She rode at an even pace along the flat and around a big bend. She journeyed slowly up a steep hill and sped down the other side. Then she headed home along a bumpy road, maintaining the same speed the whole way.

Which graph tells the story of Maggie's bike ride?





Netting Netball

Netball evolved from basketball during the 1890's. Today an estimated 2 million players are actively involved in the sport. One of the best-known netballers is South African born Irene Van Dyk. Shooting 300 goals every day has helped her achieve an astounding strike rate of 90%.



Problem

To decide who should play goal shoot, seven netball girls took 100 shots at goal. The number of successful shots made by each girl was recorded. When the girls considered their results they found the following:

The range was 71.

The mean was 40.

The data was bimodal.

Amanda was the only girl to score 35, which was the median.

Crystal's score of 82 was a clear outlier.

Sarah had the least number of successful attempts.

Renee shot 29 goals.

Dallas shot more goals than her sister Alanna.

How many goals did each girl in the team score?



Dangerous Deceit

An early credit system became evident between the 18th and 20th centuries when “tallymen” sold clothes in exchange for small weekly payments. Tallymen used a wooden stick to keep a tally of the purchases made by customers. On one side of the stick, notches in the wood represented the amount owing. Notches on the other side of the wooden stick provided a record of payments made.



<http://www.didyouknow.cd/creditcards.htm>

Problem

A fraudster has gotten hold of Andrew’s credit card. In order to use the card, a four-digit pin number must be entered. While the fraudster has remembered that all four digits are different prime numbers less than ten, he has no idea of the correct order of these numbers.

For security reasons, automatic teller machines (ATM’s) will only allow three attempts at entering a pin before the transaction is declined. What is the probability that on visiting an ATM, the fraudster will successfully withdraw money from Andrew’s account?



Dice Dilemma

In ancient times, people believed that more than just luck was involved when a dice was thrown. Instead, many believed that the gods controlled the throw of a dice. As a result, people threw dice to help make important decisions such as who should rule a country or how an inheritance should be divided.



<http://homepage.ntlworld.com/dice-play/History.htm>



Problem

Craig and his younger sister Rebecca are playing a game of 'dare' which involves tossing two six sided fair dice. Craig has set the rules for the game as follows:

When Rebecca throws: if the two numbers showing on the dice add to make a number that is greater than seven, she loses and must perform a dare.

When Craig throws: if the number three appears on either of the two dice or a 'double' (two of the same) is thrown, he loses and must perform a dare.

Who is more likely to win?



Testing Time

It is easy to confuse the terms “mean” and “median”. The **mean** is found by adding all the numbers then dividing by the number of values added. The **median**, like the median strip on a road, is found in the middle. To find the median or middle term, order the data values from smallest to largest.

If there is an odd number of data values the median is the middle number.
If there is an even number of data values there are two middle numbers.
The median is found by adding these two middle numbers and dividing by two.

Problem

A teacher scolded her 9 students, “Clearly you have not been working hard enough. The median mark for the last test was just 10%”.

A cheeky student retorted, “Actually, the class has done well. The mean test mark for the class is 50%, which is a pass mark. That’s all that matters!”

Given that both the teacher and the student’s statistics are correct, how well did each student do?



I Scream for Ice Cream

It is believed that the origins of the ice-cream cone date back to 1904. The invention happened quite by accident. An ice-cream vendor at the World's fair in St Louis, Missouri ran out of dishes to serve his ice cream. To remedy the problem, he united with the waffle vendor who rolled his waffles into ice-cream cones.



<http://www.icecream.com/funfacts/funfacts.asp?b=104>



Problem

A tub of ice cream is in the shape of a cuboid with sides $20 \times 15 \times 15$ centimetres. I coat all sides of the ice-cream block in a chocolate setting sauce. Later that evening I cut the ice cream into 36 cubes with side length 5 centimetres and place each one in a dessert bowl. I then cover the ice cream with fruit salad and cream so that the ice cream is no longer visible. The first of my party guests selects a dessert at random.

What is the probability she selects a dessert that has ice cream with no chocolate sauce?



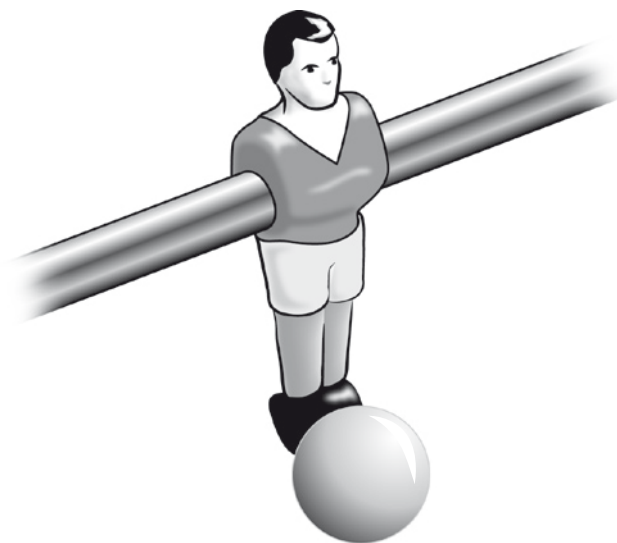


Fun with Foosball

Foosball is table soccer. As in real soccer, there are eleven 'men' on a foosball team. The aim of the game is to shoot the ball into your opponent's goal while keeping it out of your own goal. Professional foosballers battle it out at tournaments where prize money can reach hundreds of thousands of dollars.



<http://www.foosball.com/content.php?page=239>



Problem

Nicolla and Jared are playing foosball. Jared is on seven points and Nicolla is on nine points. The first player to reach 10 points will win. When one player scores a goal, the other player gets to "serve" the ball. Prior games have shown that if Nicolla serves then Jared has a 95% chance of scoring the next goal. However if Jared serves he has a 60% chance of scoring the next goal.

Given that Nicolla scored the last goal, who is more likely to win the game?



Crank Calls

The telephone was invented in 1876 by Alexander Graham Bell. Today 600 million telephone lines exist throughout the world. Despite the telephone being such an important means of long-distance communication, half the world's population has never even made a phone call!



<http://www.didyouknow.cd/fastfacts/statistics.htm>



Problem

Two naughty little boys nicknamed "Roff" and "Biff" were making some crank phone calls one afternoon by ringing telephone numbers in their town at random. Unluckily for them they accidentally telephoned their teacher who recognised their voices at once. She was furious!

"Gee, that was really bad luck!" said Biff.

Just how unlucky were the boys? Given that telephone numbers in the town have seven digits and the first three digits are always '765', what are the chances of the boys' randomly telephoning their teacher?





Steaming Statistics

Anders Celsius established The Celsius scale. Using this scale water freezes at zero degrees and boils at 100 degrees. Another temperature scale is the Fahrenheit scale where water freezes at 32 degrees and boils at 212 degrees. Atmospheric pressure affects the boiling point of water. The higher the altitude the lower the temperature at which water boils.



<http://www.didyouknow.cd/celsius.htm>



Problem

For the past week, the temperature in Waioruru (a township in the centre of the North Island, New Zealand) has increased by 1°C each day.

If I add together the seven daily temperatures the result is 140°C .

What is the median temperature for the week?



Be My Valentine

On February 14th, chocolate, flowers and gifts are exchanged between loved ones in the name of St Valentine. Debate continues to surrounds who St Valentine actually was. One popular belief is that St Valentine was a Roman priest who defied his Emperor by secretly marrying couples. Another story suggests that St Valentine sent a letter from prison to the girl he loved signed: 'from your valentine'.



<http://www.historychannel.com/exhibits/valentine/history.html>



Problem

Lynda is a cake decorator who is designing a Valentine's Day cake for a friend. She wishes to place seven chocolate hearts on the cake to represent the seven years that they have known one another. Eventually she comes up with a design that she is happy with. The design involves five rows each containing three chocolate hearts.

How has Lynda arranged the seven chocolate hearts?

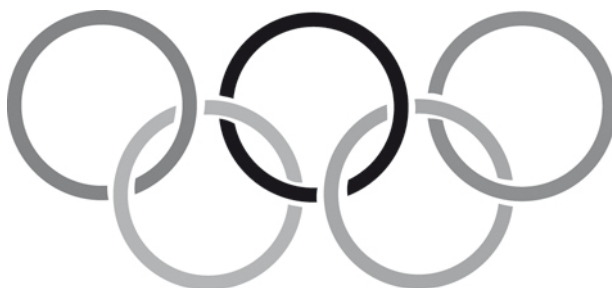


Swimming Sensations

The first Olympic Games was held in Athens, Greece in 1896. Today the Olympic programme includes almost 400 different events covering different 35 sports. One sport, which was part of the Games during the early 20th century but no longer contested, was Tug of War. The sport, which involved two teams pulling on opposite ends of a length of rope, was included as part of the athletics programme.



http://www.olympic.org/uk/sports/past/index_uk.asp



Problem

Jared, Jennifer and Lisa are champion swimmers who competed successfully at the 2004 Olympic games in Atlanta. From the clues, can you figure out which country each of the three swimmers is from, which swimming event they entered and the colour of the medal won?

Jared, the Danish swimmer, did not win the gold medal.

Norway did not enter any breaststroke swimmers.

Lisa did not qualify for the backstroke final.

The Swedish swimmer fought hard in a nail-biting finish, coming third by just 0.01 of a second.

Jennifer swam breaststroke.



April Fools

April Fool's Day is thought to have come about during the 16th century when many cultures shifted their New Years Day celebrations from the end of March or beginning of April to the 1st of January. Those folk who failed to acknowledge the new date or simply forgot were subjected to foolish pranks and became known in English as an 'April Fool' or in French as a 'poisson d'avril' (April Fish).



http://en.wikipedia.org/wiki/April_Fool's_Day



Problem

Margaret awoke one April Fool's Day to find that someone had altered all her paintings and photographs so that they were hung upside down and backwards! She quickly deduced that the culprit was either her husband Dave or one of her three sons: Kyle, Leon or Ryan. On interrogating her family, Margaret was provided with the following information:

Dave: "I didn't do it".

Kyle: "Ryan did it".

Ryan: "Dave did it".

Leon: "Ryan is lying".

If only one of the four suspects is telling the truth, who is the April Fool's day prankster?



Learning Languages

In the English language: 'Screeched' is the longest one syllable word, 'almost' is the longest word that has all its letters in alphabetical order and 'stewardesses' is the longest word that is typed with only the left hand. More than 2,700 languages exist around the world. Mandarin is the language most spoken, followed by English.



<http://www.bl.net/forwards/facts.html>



<http://www.didyouknow.cd/languages.htm>

SCREECHED

Problem

In Gillian's class, it is compulsory for students to study English and at least one other language. In her class of 25 it was found that 7 students enrolled to study German, 13 enrolled to study French and 10 enrolled to study Japanese. Two students in the class enrolled in all three languages.

If Gillian studies German and French, how many students in the class study only one language (besides English)?



La Tour Eiffel

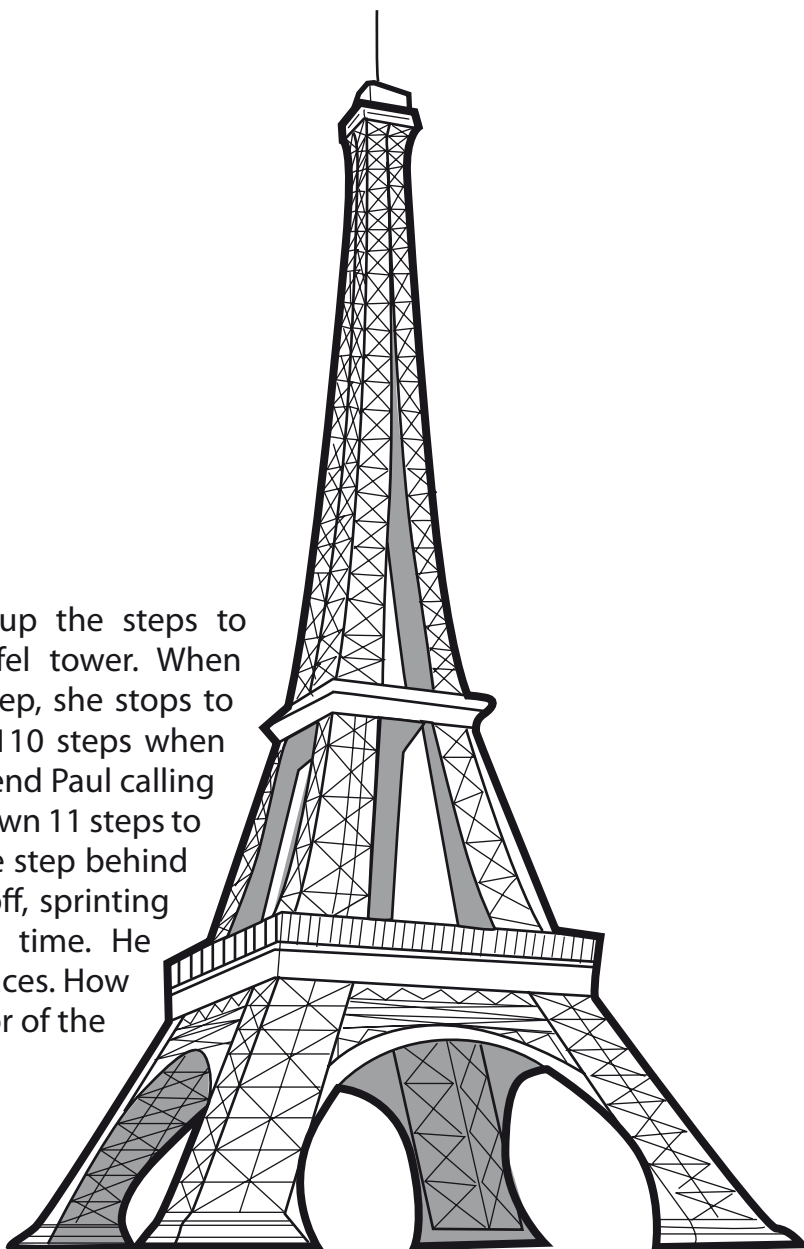
In French, the Eiffel Tower is *La Tour Eiffel*, a symbol of love and romance. At about 324 metres tall, the tower was the world's tallest structure from 1889 until 1930. Visitors to the tower can choose to climb the stairs to the first or the second level or take the elevator. To reach the top of the tower it is necessary to take a second elevator from the second level.



<http://www.tour-eiffel.fr/teiffel/uk/>

Problem

Kathryn begins walking up the steps to the first level of the Eiffel tower. When she reaches the middle step, she stops to rest. She walks a further 110 steps when suddenly she hears her friend Paul calling her. Kathryn walks back down 11 steps to meet Paul who is now one step behind her. Paul suddenly takes off, sprinting up the steps three at a time. He reaches the top after 25 paces. How many steps to the first floor of the Eiffel tower?





Four-Sided Fascination

A magic square is an array of numbers that take on the form of a square. If you add across the rows, down the columns or along the diagonals you will calculate the same number known as the 'magic sum'.



Problem

Each letter in the magic square represents a different number. Each row, diagonal and column adds to give 45. What do the letters stand for?

C	AA	BD
AD	BC	A
BA	D	AC



Difficult Dwellings

Indonesia is the largest archipelago in the world. This means that Indonesia has the largest number of islands in the world. There are 13,667 islands in Indonesia but less than half of these are inhabited. One in every ten people in the world live on an island. The largest island in the world is Greenland.

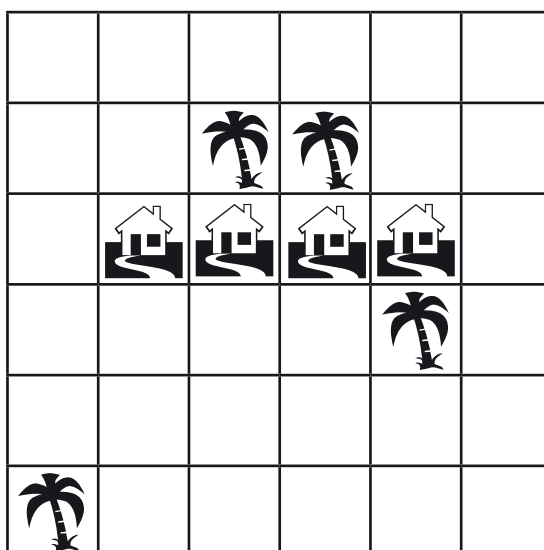


<http://www.didyouknow.cd/indonesia.htm>



Problem

A man owns an island that is perfectly square. When he writes his will he states that the island should be divided into four evenly sized and identically shaped areas for each of his four sons. Each area must contain a cabin and a palm tree. How can the man's wish be fulfilled?



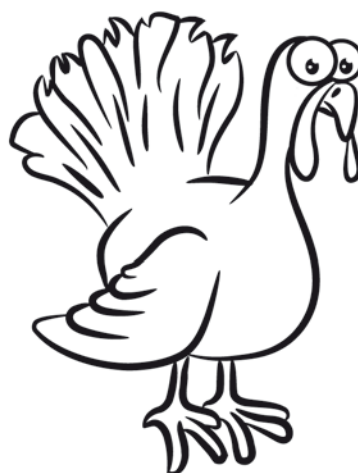


Pecan Pie

Thanksgiving is a holiday in North America to express gratitude for the bounty of the autumn harvest. People gather with their families to share a feast of food, which usually includes a large roasted turkey. For dessert, various pies are traditionally served with pumpkin pie, strawberry-rhubarb pie or pecan pie proving particularly popular.



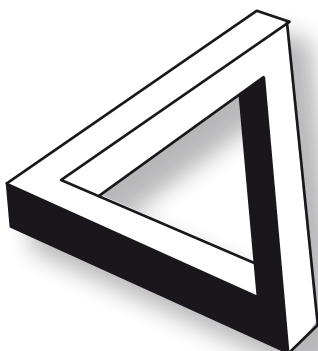
<http://en.wikipedia.org/wiki/Thanksgiving>



Problem

This year Grandma spent 6 hours preparing food for thanksgiving dinner. The pumpkin pie dessert took twice as long to prepare as the pecan pie, which took three times as long to prepare as the turkey. How long did it take Grandma to prepare the pecan pie?

creative Maths Problems - Solutions





Vertical Vertigo

Students should form and solve an equation representing a practical situation.

On which step would you expect them to pass each other?

Solution: The 636th step (counting from the bottom of the tower)

It takes Phillipa 700 seconds and Jim 1,132 seconds to complete their challenges. Thus Phillipa covers each step in 0.68 seconds while Jim covers each step in 1.10 seconds.

Let x = the step on which Phillipa and Jim pass each other (counting from the bottom of the tower). It will take Phillipa $0.68x$ seconds and Jim $1.10(1,029 - x)$ seconds to reach step x .

The relevant equation is therefore: $0.68x = 1.10(1,029 - x)$.

Solving this equation for x gives 636.

Suggestions for Teachers

Point the way: Discuss why it might be a good idea to begin by converting time taken from minutes and seconds to just seconds.

Draw a diagram: Suggest the students draw a simple diagram, labelling the point where the two meet as point 'x'.

Lead with questions: What do we know about 'x'? What information is going to be useful in working out 'x'?

Make it concrete: Ask two students to engage in a role-play of the situation for the class to help students really understand exactly what is required in solving for 'x'.



Gifts Galore

Students should investigate a pattern and report their findings concisely and coherently.

Of all the gifts that I received from my true love, which gift or gifts did I receive the most?

Solution: swans swimming and geese laying (42 of each)

In order: I received 12 partridges and drummers drumming, 22 Turtle Doves and pipers piping, 30 French Hens and Lords leaping, 36 calling birds and ladies dancing, 40 gold rings and maids milking and 42 geese laying and swans swimming.

Suggestions for Teachers

Lead with questions: Check students understand what is being asked. Ask: 'How many partridges have I received by the end of day three?' How many Turtle doves have I received by the end of day three?'

Point the way: Some students will tackle this problem the hard way. Encourage them to look for a pattern. Hint that there is a relationship between the first gift and the last gift etc.

Work together: Challenge students to work in pairs to see who can come up with the simplest, most elegant explanation of the solution.

No calculators: The emphasis should be looking for and understanding the pattern. Calculators are likely to detract from this.

Investigate further: Suppose there were many more days of Christmas. Would it always work to multiply the two middle numbers in order to find the greatest number of a particular gift? How would you know what these two numbers were? Can you find a general rule? How can you be sure it works? How many gifts did I receive in total? Can you find a rule for the total number of gifts received at the end of each day?



Money Matters

Students should find, justify and solve an algebraic formula representing a practical situation.

How much money was in John's pocket?

Solution: \$20

The four guesses add to \$86. Let the amount of money in John's pocket be represented by X then:

$$X + 10 + X - 5 + X - 7 + X + 8 = 86$$

Solving gives $X = \$20$

Suggestions for Teachers

Explain further: Most students will use a guess and check method. Examine the quicker algebraic method when going over the solution.

Point the way: Show students how to write the eldest daughter's guess in terms of X on the board ($X + 10$). Suggest they write the other guesses in terms of X for themselves.

Consolidate: To reinforce the algebra, change the total from \$86 to a new figure and ask that students attempt the problem again.



Catty Conundrum

Students should understand and apply a word formula representing a practical situation.

What human age is his cat equivalent to?

Solution: 36 years old

Russell is 23 years older than his dog and 22 years older than his cat. Therefore, Russell's cat is one (ordinary) year older than Russell's dog.

Russell's dog is equivalent in age to a 28 year old human. Therefore Russell's dog is four years old ($28 / 7$) and so his cat is five. Five years in a cat's life equates to 36 years in a human's life ($24 + 4 + 4 + 4$).

Suggestions for Teachers

Discuss: It is likely that in the first instance students will want to work out the age of their pets before solving the problem. Encourage this as it provides a practical opportunity for students to practice working with a basic word formula.

Make it abstract: Take this opportunity to introduce students to transforming the word formulae into simple algebraic formula. Practice using the algebraic formula with questions such as: "How many human years is a six year old dog?" etc.



Can It

Students should find, justify and solve a linear equation representing a given situation.

How many cans of apricots does she have?

Solution: 11 cans

Let 'x' stand for the number of cans in the first row in Barbara's pantry. Then there are x cans of asparagus, $x + 1$ cans of peaches, $x + 2$ cans of corn, $x + 3$ cans of tomatoes and $x + 4$ cans of apricots.

Solving $x + x + 1 + x + 2 + x + 3 + x + 4 = 45$, gives $x = 7$. Therefore there are $7 + 4 = 11$ cans of apricots in Barbara's cupboard.

Suggestions for Teachers

Explain further: Expect that most students will solve the problem using a guess and check method. Working the long method may help get students' appreciate the value of the quicker algebraic method.

Consolidate: Try the same problem again with different numbers.

Further activities: Challenges students to create a similar problem to swap with a friend to discuss and solve.



Give it a Try

Students should solve a problem based on relations and explain their reasoning.

How many different ways could Australia have scored 21 points?

Solution: 16 different ways

Three tries and three conversions ($5 \times 3 + 3 \times 2 = 21$)

Three tries and two penalties ($5 \times 3 + 2 \times 3 = 21$)

Three tries and two drop goals ($5 \times 3 + 2 \times 3 = 21$)

Three tries, one penalty and one-drop goal ($5 \times 3 + 3 + 3 = 21$)

Two tries, one conversion, three penalties ($5 \times 2 + 2 + 3 \times 3 = 21$)

Two tries, one conversion, two penalties, one drop goal ($5 \times 2 + 2 + 3 \times 2 + 3 = 21$)

Two tries, one conversion, one penalty, two drop goals ($5 \times 2 + 2 + 3 + 3 \times 2 = 21$)

Two tries, one conversion, three-drop goals ($5 \times 2 + 2 + 3 \times 3 = 21$)

Seven drop goals ($3 \times 7 = 21$)

Six drop goals, one penalty ($3 \times 6 + 3 = 21$)

Five drop goals, two penalties ($3 \times 5 + 3 \times 2 = 21$)

Four drop goals, three penalties ($3 \times 4 + 3 \times 3 = 21$)

Three drop goals, four penalties ($3 \times 3 + 3 \times 4 = 21$)

Two drop goals, five penalties ($3 \times 2 + 3 \times 5 = 21$)

One-drop goal, six penalties ($3 \times 1 + 3 \times 6 = 21$)

Seven penalties ($3 \times 7 = 21$)

Suggestions for Teachers

Discuss: Talk about the different ways points can be scored in rugby to be sure there is no confusion.

Break it up: It is important to be systematic. Consider using a table to organise findings. Work through all the possible combinations from most possible tries scored (three) to least possible tries scored (zero).

Make it abstract: Discuss the equation that is being followed in finding each of the 16 combinations of tries (t), conversions (c), penalties (p) and drop goals (d)? What constraints are necessary (i.e. $5t + 2c + 3p + 3d = 21$ where $c \leq t$)?



Biking Bliss

Students should interpret graphs representing a simple everyday situation.

Which graph tells the story of Maggie's bike ride?

Solution: Graph C

Graph C is the only graph that could possibly correspond to the story of Maggie's bike ride. Note that Graph A is inaccurate because 'speed' must start and end at zero.

Suggestions for Teachers

Discuss: What is the best graphing method for recording Maggie's bike ride? Which is the easiest to interpret? Why do you think this might be?

Investigate further: List the graphs A – F. What is misleading about each one? What features about the graph show you that it is wrong?

Further activities: Challenge students to create their own graph to represent an everyday situation. Swap with a friend who must try to retell the story illustrated by the graph.



Netting Netballs

Students should be knowledgeable about the distinctive features of data displays.

How many goals did each girl in the team score?

Solution: Crystal – 82, Dallas – 47, Kate – 47, Amanda – 35, Renee – 29, Alanna – 29, Sarah – 11

Crystal's score of 82 was a clear outlier therefore she alone achieved the top score.

The range was 71. As Sarah scored the least number of goals her score must have been $82 - 71 = 11$.

Amanda was the only girl to score 35.

The data is bimodal and Alanna scored fewer goals than Dallas. Thus Alanna must share Renee's score of 29.

The mean was 40, so the total number of goals shot by all seven girls is $40 \times 7 = 280$. Subtracting Crystal, Sarah, Amanda, Renee and Allawah's scores leaves 94. Because the data is bimodal, Kate and Dallas must both have scored 47 goals ($94/2$).

Suggestions for Teachers

Break it up: Adopt a systematic approach, using and eliminating pieces of information bit by bit. List the numbers 1-7 (best to worst shooters) and gradually fill in the appropriate players name.

Lead with questions: The word 'bimodal' is a potential stumbling block. Help students work out the meaning for themselves by breaking the word into two parts.

No calculators: The use of calculators is not necessary for this problem.



Dangerous Deceit

Students should calculate the theoretical probability of a given event occurring.

What is the probability that on visiting an ATM, the fraudster will successfully withdraw money from Andrew's account?

Solution: $3/24 = 1/8 = 0.125$

The 24 possibilities are:

2,357	3,257	5,237	7,235
2,375	3,275	5,273	7,253
2,537	3,527	5,327	7,325
2,573	3,572	5,372	7,352
2,735	3,725	5,723	7,523
2,753	3,752	5,732	7,532

As the Fraudster is allowed up to three attempts at entering the correct PIN number, the probability that he will successfully withdraw money is $3/24$.

Suggestions for Teachers

Point the way: This question assumes that students are familiar with prime numbers less than ten.

Lead with questions: Discuss adopting a systematic approach: How can we be sure we have covered all possibilities?

Draw a diagram: Draw a tree diagram to represent the situation.

Make it concrete: Act out a role-play. One person is Andrew and knows the number. Class members take turns being the fraudster.

Worktogether: Suggest working individually to write down all possible numbers before swapping with a friend to check all numbers have been covered.



Dice Dilemma

Students should find all possible outcomes for a sequence of events.

Who is more likely to win?

Solution: Craig.

There are 36 possible outcomes from throwing two dice.

Rebecca has a $21 / 36$ chance of losing as there are 23 possible ways of throwing two dice so that the two upturned numbers add to make a total greater than seven.

Craig has a $16 / 36$ chance of losing as there 16 possible ways of throwing two dice so that at least one three appears or a double is thrown.

Therefore Craig is more likely to win the game.

Suggestions for Teachers

Point the way: Suggest students begin by writing out all the possible outcomes from throwing two fair six sided dice.

Lead with questions: What is meant by 'is the game fair'? How can we check if something is fair? Is it important that the dice they are playing with are fair? Why or why not? How could we check to see if a dice was truly fair?

Discuss: Talk about the terms "highly likely", "likely", "50:50", "unlikely" and "highly unlikely" in relation to the dice game and Rebecca and Craig's chances of winning.

Investigate further: Run a simple experiment with the class to check whether a six sided dice is fair. How could Craig have devised a dice game so that each player had an equal chance of winning / losing?



Testing Time

Students should evaluate other's interpretations of data displays.

How well did each student do?

Solution: Five students in the class performed very poorly. Four students attained perfect scores.

There are nine students in the class. In order for the class to have attained a median mark of 10% and a mean mark of 50%, the students' marks must be as follows: 10, 10, 10, 10, 10, 100, 100, 100, 100. Therefore the data is bimodal and just over half the class attained very poor marks while just under half the class achieved perfect scores.

Suggestions for Teachers

Lead with questions: What is the lowest percentage mark and what is the highest percentage mark that can be achieved? How do we calculate the mean? How do we work out the median? What can we immediately determine about four of the students' marks?

Discuss: Do you think mean or median is better for describing the students' results? Explain. Can you think of a better statistic to describe the results? Explain. Do you think these test results very realistic? Why or why not?

Further activities: From the information we are able to ascertain all the marks that were attained in the test. What are they? Graph the marks with frequency on the y-axis and test score on the x-axis. What do you notice about the shape of the distribution of scores?



I Scream for Ice Cream

Students should calculate the theoretical probability of a given event occurring.

What is the probability she selects a dessert that has ice cream with no chocolate sauce?

Solution: $2/36 = 1/18$

Of the 36 ice-cream cubes, two will contain no chocolate sauce because they are positioned in the very centre of the ice-cream block. The probability of selecting a dessert containing no sauce is therefore $2/36$.

Suggestions for Teachers

Lead with questions: How many cubes have 3 sides coated in sauce? How many have 2 sides? One side? No sides?

Discuss: Suppose the ice cream was visible. What affect might this have on the probability?

Draw a diagram: Suggest drawing a 3-dimensional diagram of the cuboid and the divisions when it is cut into 36 smaller cubes.



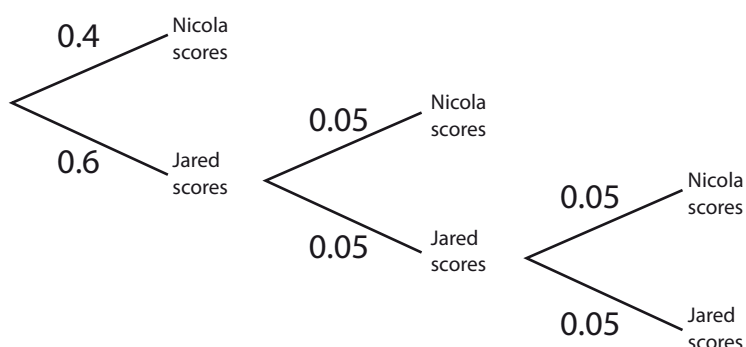
Fun with Foosball

Students should calculate probability by finding all possible outcomes for a sequence of events using a tree diagram.

Who is more likely to win the game?

Solution: Jared is more likely to win (with probability 0.5415)

In order for Jared to win, he must score the next three goals (in a row). The tree diagram shows all the possibilities:



The probability that Jared will win is therefore $0.6 \times 0.95 \times 0.95 = 0.5415$. The probability that Nicolla wins is $1 - 0.5415 = 0.4585$.

Suggestions for Teachers

Draw a diagram: Draw a tree diagram to show all the possible outcomes for how the game will finish. Write the probabilities on the branches.

Lead with questions: What happens if Nicolla scores another goal? What needs to happen in order for Jared to win the game?

Investigate further: Do you think the probabilities outlined in the question are accurate given that Nicolla has scored nine goals and Jared has scored seven goals?



Crank Calls

Students should determine the theoretical probability of a given event occurring.

What were the chances of the boys telephoning their teacher?

Solution: The boys were extremely unlucky! The probability of them telephoning their teacher is $1/10000$

The first three digits of the telephone number are always '765' but the fourth, fifth, sixth and seventh digits can be any number between 0-9 inclusive (ten possibilities).

Therefore there are $10 \times 10 \times 10 \times 10 = 10,000$ possible telephone numbers the boys could have called. One of these numbers belongs to their teacher and so the probability is $1 / 10,000$.

Suggestions for Teachers

Point the way: Think about how many choices you have available before dialling each number.

Discuss: What is meant by the term random? The probability of telephoning their teacher is extremely unlikely. Do you think that the boys were truly telephoning numbers at random? Is there another explanation?



Steaming Statistics

Students should apply their understanding of means and medians.

What is the median temperature for the week?

Solution: 20°C

The mean of the seven numbers is 20°C degrees Celsius as $140 / 7 = 20$. As the temperature has increased by 1°C each day, the seven data values will be symmetrical about the mean. Therefore the median will be equal to the mean.

Suggestions for Teachers

Lead with questions: What do you know about the mean temperature for the week? Could you use the mean temperature to help you calculate the median?

Investigate further: The mean and the median for the seven numbers are the same. Does this always occur? What circumstances lead the mean and the median to be equal?

No calculators: Calculators are likely to encourage students to use trial and error rather than apply their statistical knowledge.

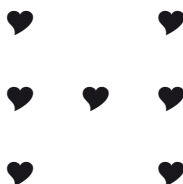


Be My Valentine

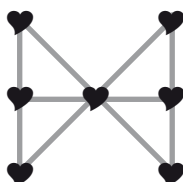
Students should seek a solution by following instructions and trying alternatives.

How has Lynda arranged the seven chocolate hearts?

Solution:



The arrangement has five rows each containing three hearts:



Suggestions for Teachers

Lead with questions: If students seem stuck ask: Do rows need to be horizontal or vertical?

Further activities: For fast finishers: Suppose Lynda has 9 chocolate hearts for her cake, what is the maximum number of rows of three that she could make? (Answer is 10.)



April Fool's

Students should be able to make simple deductions.

Who is the April fools prankster?

Solution: Dave

If Kyle, Leon or Ryan committed the crime then more than one suspect would be telling the truth.

However, if Dave committed the crime then:

Dave is lying, Kyle is lying, Ryan is telling the truth and Leon is lying.

As only one suspect is telling the truth, Dave is the April fools prankster.

Suggestions for Teachers

Point the way: Suggest drawing up a table similar to the table below and filling in each box with 'no' or 'yes'.

Assumption	Kyle is Lying	Leon is lying	Ryan is lying	Dave is lying
Kyle is the culprit				
Leon is the culprit				
Ryan is the culprit				
Dave is the culprit				



Swimming Sensations

Students should critically follow a chain of reasoning.

Which country, swimming event and medal belongs with each swimmer?

Solution: Lisa – Butterfly – Norway - Gold Medal; Jared – Backstroke – Denmark – Silver Medal; Jennifer – Breaststroke – Sweden – Bronze Medal.

Jared, the Danish swimmer did not win gold, nor did he win bronze (because the Swedish swimmer won bronze) therefore Jared won the silver medal.

Jennifer swam breaststroke. Therefore Jennifer is not from Norway (as Norway did not enter any breaststroke swimmers) and she is not from Denmark (as Jared is from Denmark) so she must be from Sweden. Thus Jennifer won the bronze medal.

This means that Lisa is from Norway and she won gold. Lisa did not qualify for the backstroke final and as Jennifer swam breaststroke, Lisa must have swum butterfly. Therefore Jared swam backstroke.

Suggestions for Teachers

Point the way: Suggest students draw a chart like the one below, to fill in, as they unravel the clues:

	Back	Fly	Breast	Norway	Denmark	Sweden	Gold	Silver	Bronze
Lisa									
Jared									
Jen									
Gold									
Silver									
Bronze									
Norway									
Denmark									
Sweden									



Learning Languages

Students should organise and interpret information

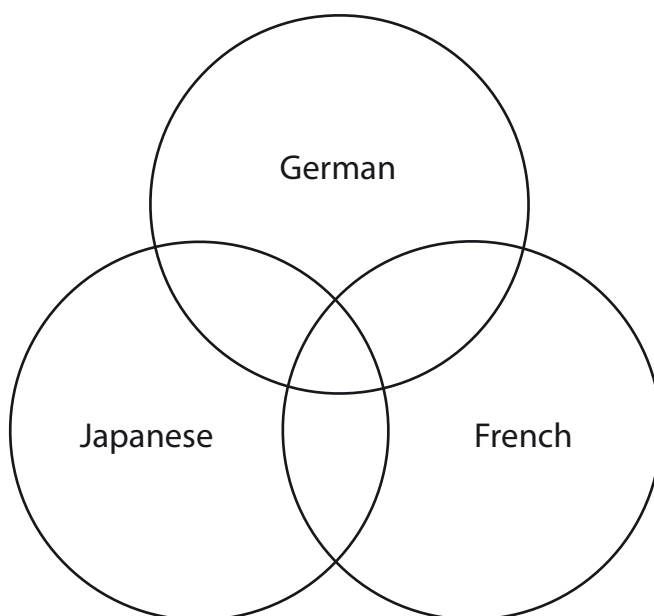
How many students in the class study only one language?

Solution: 22 students

Adding together the number of students enrolled in each of the three languages gives 30 language enrolments. However there are only 25 students in the class. This means that five of the thirty language enrolments belong to students studying more than one language. Four of these five enrolments belong to the two students learning all three languages. The fifth enrolment belongs to Gillian. As the five 'extras' are accounted for, these three students are the only students enrolled in more than one language. This leaves $25 - 3 = 22$ students in the class studying just one language.

Suggestions for Teachers

Point the way: A Venn diagram approach might be useful i.e.





La Tour Eiffel

Students should demonstrate logic in following a systematic chain of reasoning.

How many steps are there?

Solution: 347

Paul reaches the top after 25 paces. As $25 \times 3 = 75$, Paul must have been 75 steps from the top of the staircase when he left Kathryn. Kathryn was one step ahead of him at this time and therefore must have been 74 steps from the top.

Kathryn had walked back down 11 steps to meet Paul. Therefore Kathryn must have been 63 steps from the top step when she heard Paul calling her.

Kathryn walked up 110 steps from the middle step to get to the step that is 63 from the top. Therefore the middle step must be the 174th step and so the total number of steps is 347.

Suggestions for Teachers

Point the way: Hint that working backwards might be a good strategy.

Draw a diagram: Suggest drawing a simple sketch to help clarify things.

No calculators: This problem provides good practice at simple mental arithmetic.



Four-Sided Fascination

Students should be able to make a series of simple deductions.

What do the letters stand for?

Solution:

5	22	18
28	15	2
12	8	25

Consider the diagonal starting at the bottom left and moving to the top right. The three numbers begin with the same digit (B). In order for them to add to 45, B can only possibly be equal to 1.

Consider now, the diagonal starting at the top left and moving to the bottom right. All the numbers end with the same digit (C). As $B = 1$, the only possibility is that $A = 2$ and $C = 5$. This means that $D = 8$.

Suggestions for Teachers

Point the way: Check that students realise that two digits placed together represents a two-digit number (not two numbers being multiplied together).

No calculators: This problem provides good practice at mental arithmetic.

Lead with questions: Where would a good starting place be? Why? How can you tell immediately what digit the letter B must represent?

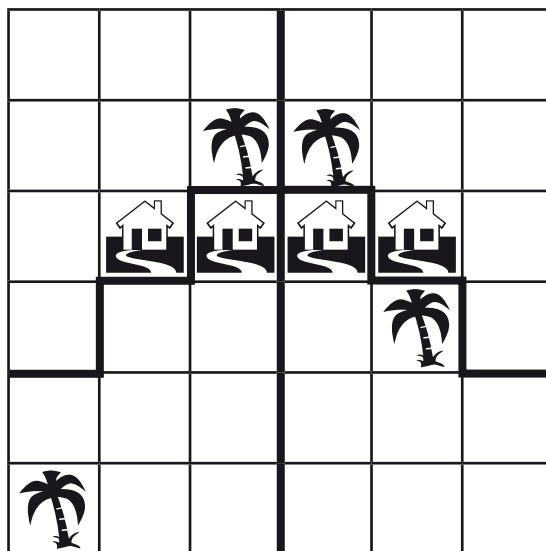


Difficult Dwellings

Students should seek a solution by trying alternatives and thinking flexibly.

How can the man's request be fulfilled?

Solution:



The solution shows the property has been evenly divided into four identically sized and shaped sections as required.

Suggestions for Teachers

Lead with questions: Each property has to be the same size. What can you immediately deduce from this?

Draw a diagram: In order to try different alternatives it is advisable that students draw the diagram of the island for themselves to work from.



Pecan Pie

Students should use logic in order to understand and interpret information and seek a solution.

How long did it take Grandma to prepare the pecan pie?

Solution: 108 minutes

Let the preparation time for the turkey = X , then the preparation time for the pecan pie = $3X$ and the preparation time for the pumpkin pie = $6X$.

As $10X = 360$ minutes (6 hours), $X = 36$ minutes and $3X = 108$ minutes.

Suggestions for Teachers

Point the way: Hint working backwards from the preparation time for the turkey.

Explain further: Most students will use a "guess and check" method. Discuss the algebraic method when going through the solution.

Consolidate: Try the problem again using different figures.